ITCS 312: Automata and Formal Languages

Exam 2, Second semester 2015/2016, Form: $\boxed{A}$

Name: KEY

Student Number: ____________

Section: ____________

Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

F The best method for answering the membership question is by using exhaustive search.

F The language $L = \{a^n b^n c^m : n < 100, m > 0\}$ is not regular.

T The language $L = \{w \in \{a, b\}^* : 2n_a(w) < 4n_b(w)\}$ can be accepted by an NPDA.

T The grammar $S \to aSb|abS|aS|a|b$ is ambiguous. Try deriving abab

F If $L_1 \cup L_2$ is regular then L_1 is regular or L_2 is regular.

T There are languages which have no un-ambiguous grammar, and such languages are called inherently ambiguous.

T Every regular language is also context-free.

T The language $L = \{a^{2n} b^{3m} c^{2n} : n, m \geq 0\}$ is context-free.

T NPDAs machines accept context-free languages as well as regular languages.

T The language $L = \{w : n_a(w) = 2n_b(w)\}$ is not regular.

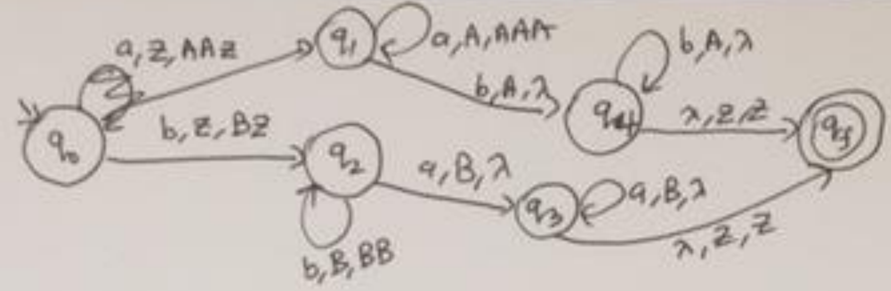
Section 2. (5 points each)

1. Prove or disprove that the following language $L = \{a^n b^m a^k : m = n + k\}$ is context-free.

$S \to AB$

$A \to aAb \mid \lambda$

$B \to bBa \mid \lambda$



2. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$ be an NPDA with

$$
\begin{aligned}
\delta(q_0, a, Z) &= \{(q_1, AAZ)\} \\
\delta(q_0, b, Z) &= \{(q_2, BZ)\} \\
\delta(q_1, a, A) &= \{(q_1, AAA)\} \\
\delta(q_1, b, A) &= \{(q_4, \lambda)\} \\
\delta(q_4, b, A) &= \{(q_4, \lambda)\} \\
\delta(q_4, \lambda, Z) &= \{(q_f, Z)\} \\
\delta(q_2, a, B) &= \{(q_3, \lambda)\} \\
\delta(q_2, b, B) &= \{(q_2, BB)\} \\
\delta(q_3, a, B) &= \{(q_3, \lambda)\} \\
\delta(q_3, \lambda, Z) &= \{(q_f, Z)\}
\end{aligned}
$$

(a) What is the value of Q, Σ, and Γ?

$Q = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma = \{a, b\}$
$\Gamma = \{Z, A, B\}$

(b) What is the language accepted by M assuming $F = \{q_f\}$?

$L = \{a^n b^{2n}\} \cup \{b^n a^n\}$

3. Convert the following grammar to Greibach normal form.

$$
\begin{aligned}
S &\to aaSbb | aA | bbB \\
A &\to a | aa | aaa \\
B &\to b
\end{aligned}
$$

$X \to b$
$Y \to a$

$S \to aYSXX \mid aA \mid bXB$
$A \to a \mid aY \mid aYY$
$B \to b$

GNF

4. Prove that $L = \{b^n a^p b^{n \times p} : n > 0, p > 0\}$ is not regular.

Let $w = b^m a^m b^{m^2}$ $\quad w \in L$ and $|w| \geq m$.

Let $w = xyz$.

Since $|xy| \leq m$ and $|y| \geq 1$,

$y = b^k \quad 1 \leq k \leq m$.

$w_0 = b^{m-k} a^m b^{m^2} \notin L$ because $k \geq 1$.

$\therefore$ L is not regular by the pumping lemma.

5. Given the following language, show that it is context-free by constructing an NPDA that accepts it.

$$L = \{w \in \{a, b\}^* : n_a(w) = n_b(w) + 1\}$$